The Regularized Ten Body Problem

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ABSTRACT

It has been shown in the paper "The Invariant Plane" that a higher order of symmetry of the Solar System is obtained by transforming into a new coordinate system. This transformation results in two distinct schemes for the planets which are pi radians out of phase. Two planets, Venus and Pluto, are out of phase with all the others, and these are the only two planets that exhibit retrograde rotation. It is therefore logical to postulate that the entire solar system, in some context, can be regularized into a Three Body Problem: the sun, plus the two sets of planets distinguished by the coordinate transformation into the Invariant Plane context. A helical equation is assumed for the combined solution to the Ten Body Problem and, when substituted into the formula for the regularized Three Body Problem, there results a closed form solution. Having thus proven the solution, the analysis validates a mechanical analog to the imaginary number, "j" as a rotational motion with a "complex" ordinate of the Invariant Plane itself.

Mechanical Equivalence

It has been shown in the paper "The Invariant Plane" that a higher order of symmetry of the Solar System is obtained by transforming into a new coordinate system. This plane is represented by a slope that varies sinusoidally around 360° of heliocentric longitude, and an intercept that also varies sinusoidally around the celestial sphere. These form a 3D plane that is flat with the exception of a distortion at the origin, in the vicinity of the intense gravitational field of the sun. This is identified as the Invariant Plane.

A straight line can be made to behave in just this manner by considering that a point on a rotating circle traces out a sinusoidal wave form in 2D space. In the current case there are two sinusoidal motions occurring simultaneously, acting upon a straight line. So, attaching one end of a straight line to a circle centered at the origin, and attaching the line (in a sliding link) to another circle at approximately 2 AU, it is possible to cause the line to behave as noted. As both circles rotate, in one complete revolution with 360° of heliocentric longitude, they form the Invariant Plane.

A formal development of the wave generated by this mechanism in 3D space is found in what are known as the Cochoids of Nichomedes which have the parametric equations:

$$x = a + \cos(t) \qquad (1)$$
$$y = a*\tan(t) + \sin(t)$$

Geometrically, these are created by following the motion of one point on the circumference of a circle as it rolls along a flat surface. In 3D the result is a helix. This wave form is able to exhibit two sets of objects 180° out of phase, as required by the conditions of the problem.

The Invariant Plane coordinate transformation has regularized the Ten Body Problem, in a geometrical sense, into a Three Body Problem. Analytically, the parametric equations (1) have done the same thing. Consequently, these equations should form a solution to a Regularized Three Body. The mathematical development in Figures 1 and 2 substitutes equations (1) into the so-called Szebehely Equation for the Regularized Elliptical Three Body Problem. This works because of the geometry of the mechanical model, which determines the value of a in the parametric equations. This, in turn, introduces the complex number "j" into the function which, if accepted as is, leads finally to a closed solution to the Szebehely Equation. The equations (1) regularize the Ten Body Problem into a Three Body Problem.

The significance of "j" in this analysis is apparently the same as it's conceptualization in engineering dynamics: it implies rotation; here, the rotation is of the body itself, which is contrary to the rotation of the bodies in the adjacent scheme. The complex ordinate about which this rotation occurs is the axis of rotation of the constituent planet(s).

Figure 1

Mathematics

To confirm the data with function (1). First, develop the radius r in the cartesian coordinate system

$$r^2 = x^2 + y^2 \tag{2}$$

$$= x^2$$

therefore, $r \approx x \approx a + \cos t$ (3)

This can be substituted into the function for y,

$$\begin{aligned} y &= a \tan t + \sin t \\ &= \sin t \,(a/\cos t + 1) \\ &= \tan t \,(a + \cos t) \end{aligned} \tag{4}$$

Where a = 0.05 in the illustration, but is actually << 0.05. Thus,

$$(a + \cos t) = j \cos t \tag{5}$$

This will result in rotation about the central point, such that motion is above the system plane. This introduction of the constant "j" is analogous to the development of the orbital model in quantum mechanics. The significance of this assumption, and so the justification for it, will be evident later in the analysis.

$$\begin{aligned} y &= j \sin t \\ x &\cong a^2 + \cos t \text{ (as before)} = j \cos t \\ x &= j \cos t \quad , \quad \dot{x} = -j \sin t \\ y &= j \sin t \quad , \quad \dot{y} = j \cos t \end{aligned} \tag{6}$$

Now these values are substituted into the Szebehely equation for the restricted three body problem,

$$\ddot{y} = \frac{(1 + y')^2}{\Lambda^2} \left(\Lambda_y - \dot{y} \Lambda_x \pm 2 \sqrt{1 + y'^2} \right) \tag{7}$$

from which the constant Gamma is defined as

$$\Lambda^2 = \dot{x}^2 + \dot{y}^2 \tag{8}$$

Figure 2

substituting,

$$\Lambda^2 = -\sin^2 t - \cos^2 t = -1 \quad \therefore \quad \Lambda = j$$

$$\Lambda_x = \Lambda_y = 0$$

$$\text{because} \quad \frac{\partial}{\partial x}(\text{const}) = 0$$

Thus, the Szebehely equation explains the variation from the "system plane wave," defined by the prolate cycloid rotated about the central axis. Substituting back into that equation,

$$-j\sin t = \ddot{y} = \frac{(1+j\cos t)^2}{-1}\left(\pm 2\sqrt{1-\cos^2 t}\right) \tag{9}$$

$$-j\sin t = -(1+j\cos t)^2\,(\pm 2\sin t)$$

$$+j = -(1+j\cos t)^2\,(\pm 2)$$

$$\pm\frac{j}{2} = -1 - 2j\cos t + \cos^2 t$$

Which must satisfy two levels of equality, for the real and the imaginary parts.

(1) $\pm\frac{j}{2} + 2j\cos t = 0$

$$2\cos t = \pm \tfrac{1}{2}$$

(2) $1 = \cos^2 t$

$$\emptyset = t$$

where t = 0 is an assumption made for the development of the Szebehely equation.

This fairly reasonable result came because of the one assumption made earlier concerning "j." The dynamics this introduces to the analysis are investigated in the following section, in a heuristic fashion.