The Invariant Plane

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ABSTRACT

This report seeks an Invariant Plane for the Solar System, one within which the motion of all the planets is symmetric. Such a plane is developed as a least squares fit, and it is shown to be symmetric. This symmetry is investigated in three ways: in the XY plane of the new coordinate system, perpendicular to this plane in the Z direction, and perpendicular to the X direction. The motion of the planets is shown to be symmetric in all three schemes. Thus, the validity of the Invariant Plane as developed is qualified.

The concept of an invariant plane is that of a special coordinate system about which the motions of the constituent bodies is symmetric. In the case of the solar system this invariant plane is sought by a study of the "Inverse Problem of Celestial Mechanics." The data for this study are the known positions of the planets. These data are analyzed in a simple sequence in Figure 1, as follows:

(1) The position of each planet at 0° heliocentric longitude is calculated from the orbital elements.
(2) A linear least squares fit is made of these nine data points, giving a slope and intercept
(3) This is repeated at 10° increments through 360° of longitude.

The slope (Figure 2) of this line varies sinusoidally, which thus defines a flat XY plane in 3D space. The intercept (Figure 3) also varies sinusoidally. Presumably this creates a relativistic warping of space in the immediate vicinity of the origin, caused by the intense gravitational force of the sun at the center of the heliocentric sphere. This notion is beyond the scope of the current paper and will be the subject of another report.

The true measure of this new plane in space is how symmetric the motion of the planets is with respect to it. This motion should be uniformly sinusoidal in both the Z-direction and the Y-direction. These are studied separately.

The motion perpendicular to the coordinate system plane is in the Z-direction (Figure 4). All planets (excepting Mercury, Venus, and Pluto) are sinusoids exactly in phase, with the same leading phase angle. Venus and Pluto are exactly pi radians out of phase; they also exhibit retrograde rotation, so being out of phase with the other planets in prograde rotation is a logical consequence. Also, adjusting the intercept to be fixed at the center of the Sun (which itself moves around the barycenter of the Solar System in a small orbit) puts Mercury in phase with the other planets. Again, this exception will be the subject of a separate study.

The motion in the Y-direction of the coordinate system is not as revealing (Figure 5), exhibiting sinusoidal motion as expected due to the elliptically of the orbits. A plot of the complete motion in the YZ plane is more instructive (Figure 6). It shows that while the motion perpendicular to the plane is uniformly above and below that plane, it is also nearly perpendicular for every planet except Pluto. This means the new coordinate system acts so as to reduce the conic section of each planet's orbit into a helix in 3D space; or a circle when projected down onto the Invariant Plane.

Conclusion

A study was done to seek a special coordinate system by which the motion of the planets is "regularized" or made uniform. This was done successfully, the motion being - with a few logical exceptions - uniformly consistent in-plane and perpendicular the plane. Further, the combination of these motions approaches an even simpler system, a helix.

Figure 1 - the Source Code

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C
C     The Inverse Problem of Celestial Mechanics
C     by William H. Clark II, P.E.
C
      PROGRAM SEROS
      implicit REAL*8 (a-h,o-z)
      REAL*8 Elemts(5,9),Hx(40),Celestex(10,40),Celestey(10,40),
     +       LSfit(40,2),Bary(10,40)
      OPEN (6,FILE='a.out')
    1 FORMAT (1X,2E8.4)
      pi = DACOS(-1.D0)
C
C     Read orbital elements into an array for the nine planets; source:
C     Celestial Basic: Astronomy On Your Computer
C     by Eric Burgess, Fellow Royal Astronomical Society
C     (1960 Epoch) published by Sybex
C
      DATA Elemts / 0.387100,0.079740,2.735140,0.122173,0.836103,
     +              0.723300,0.005060,3.850170,0.593410,1.331680,
     +              1.000000,1.017000,3.339290,0.000000,0.000000,
     +              1.523700,0.141704,1.046560,0.031420,0.858702,
     +              5.202800,0.249374,1.761880,0.019720,1.745330,
     +              9.538500,0.534156,3.125700,0.043633,1.977458,
     +             19.182000,0.901554,4.490840,0.013960,1.288050,
     +             30.060000,0.270540,2.334980,0.031416,2.291620,
     +             39.440000,9.860000,5.231140,3.001970,1.918120 /
C
C     Read data into heliocetric longitude array in 10 degree
increments
C     where Hx(1) = 0 degrees; Hx(35) = 350 degrees
C     (calculation units are in radians)
C
      Hstep = 0.D0
      DO 10 n=1,36
           Hx(n) = Hstep
           Hstep = Hstep + pi/18.D0
   10 END DO
C
C     Calculate in heliocentric coordinate system at each Hx value::
C     x-coordinate, Celestex(planet,Hx) = distance from sun
C     y-coordinate, Celestey(planet,Hx) = distance from ecliptic
C
      DO 30 m = 1,9
C           write (6,*) m
           DO 20 n=1,36
                Celestex(m,n) = Elemts(1,m) + Elemts(2,m)*
     +                              DSIN(Hx(n) - Elemts(3,m))
                Celestey(m,n) = Elemts(4,m)*
     +                              DSIN(Hx(n) - Elemts(5,m))
C           write (6,*) Celestex(m,n), Celestey(m,n)
   20      END DO
   30 END DO
```

```
C
C     Calculate the simple (linear) least squares fit at each Hx value
C     LSfit(Hx,1) = slope and LSfit(Hx,2) = intercept
C     Correlation ?
C
      DO 50 n = 1,36
           sumx  = 0.D0
           sumy  = 0.D0
           sumxx = 0.D0
           sumyy = 0.D0
           sumxy = 0.D0
           DO 40 m = 1,9
                sumx  = sumx  + Celestex(m,n)
                sumy  = sumy  + Celestey(m,n)
                sumxx = sumxx + Celestex(m,n)*Celestex(m,n)
                sumyy = sumyy + Celestey(m,n)*Celestey(m,n)
                sumxy = sumxy + Celestex(m,n)*Celestey(m,n)
   40      END DO
           LSfit(n,1) = (9.D0*sumxy - sumx*sumy)/
     +                  (9.D0*sumxx - sumx*sumx)
           LSfit(n,2) = sumy/9.D0 - LSfit(n,1)*(sumx/9.D0)
      write (6,*) LSfit(n,1), LSfit(n,2)
   50 END DO
C
C     Determine the position of the planets relative to these 36 lines
C     (the x-distance from the sun is radial, so it is unchanged)
C
      DO 70 n = 1,36
           DO 60 m = 1,9
                Bary(m,n) = Celestey(m,n) -
     +                      LSfit(n,1)*Celestex(m,n) + LSfit(n,2)
   60      END DO
   70 END DO
C
C     The coordinates of each planet versus the Invariante Plane are
C     (where n = heliocentric longitude in degrees) n = 1,36
C     Celestex(planet,n) ; Bary(planet,n)
C     Read these to an external file so they can be studied in Excel
C
      DO 90 m = 1,9
           write (6,*) m
           DO 80 n = 1,36
C                WRITE (6,*) Celestex(m,n), Bary(m,n)
   80      END DO
   90 END DO
      END PROGRAM
```

Figure 2 - Slope

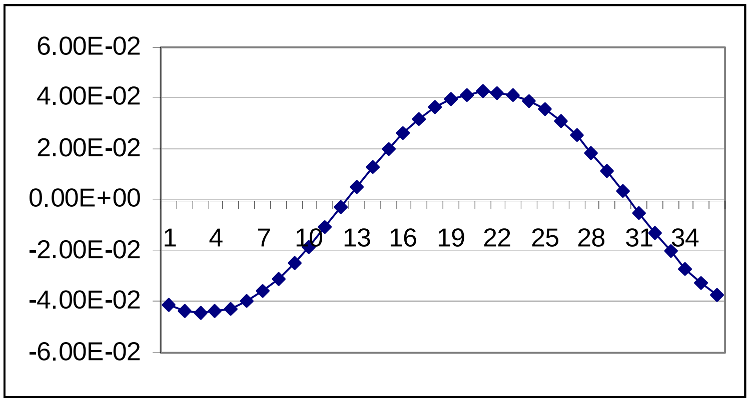

Figure 3 - Intercept

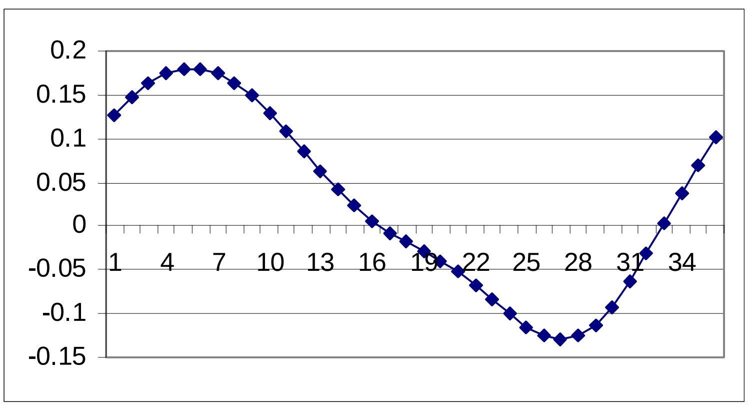

Figure 4 Motion in the Y-direction (in the Invariant Plane)

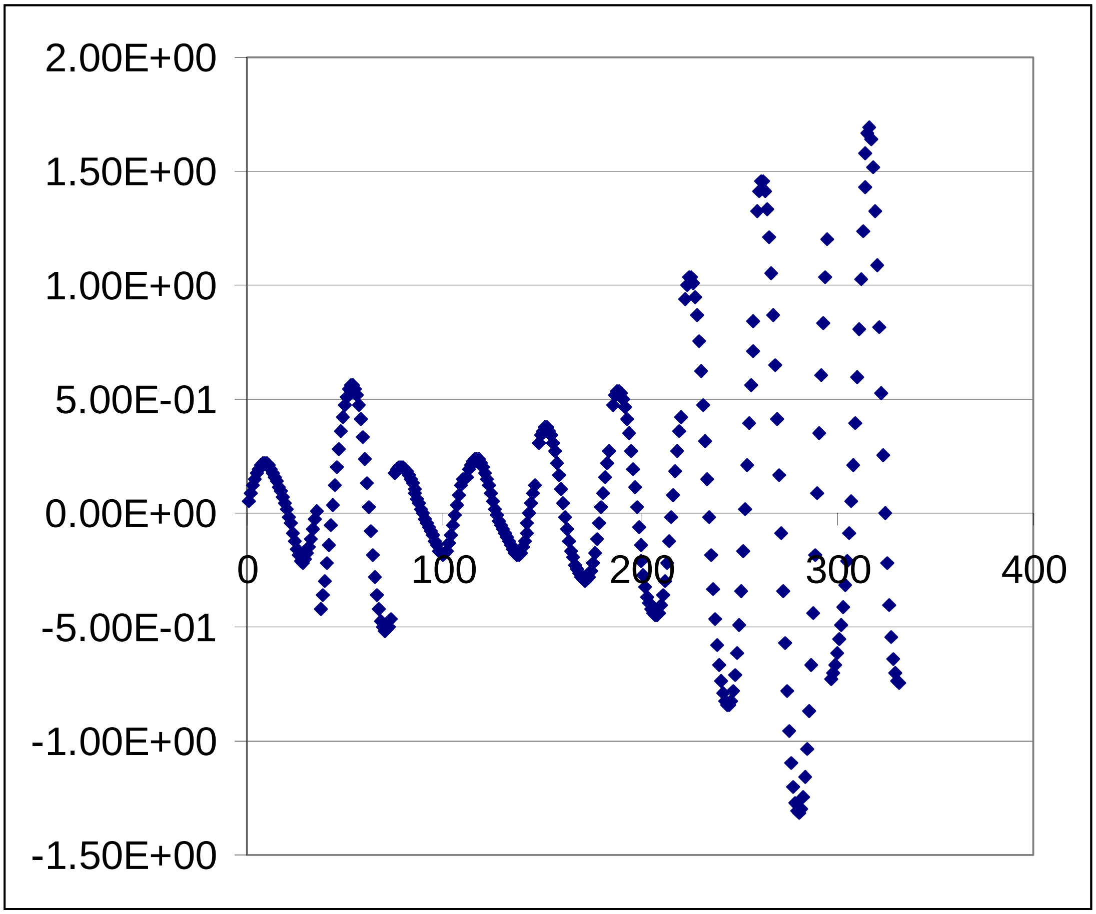

Figure 5 Motion in the Z-Direction (perpendicular to the Invariant Plane)

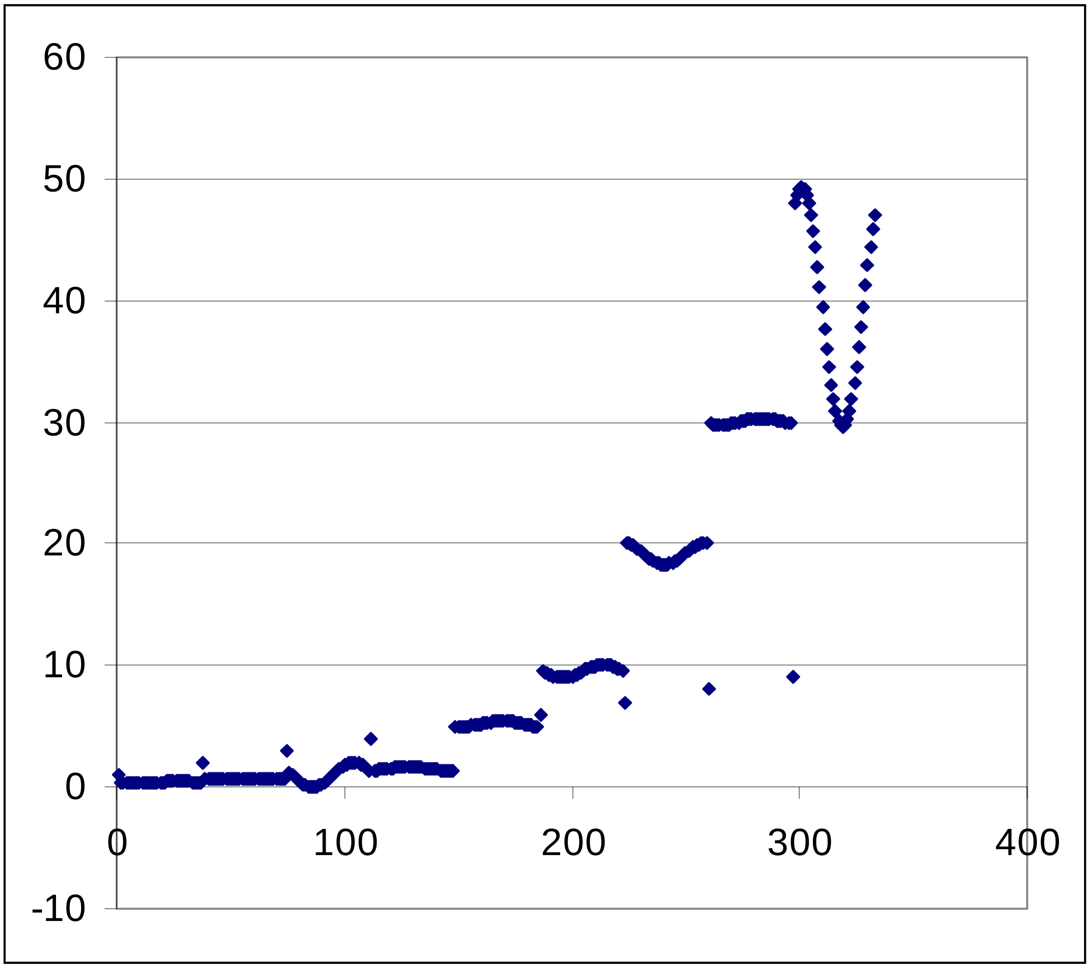

Figure 6 Motion Perpendicular to the X-Direction