

The Gravity Ellipsoid

By

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ABSTRACT

Having established a new invariant plane for the solar system, the fundamental frequency is designated the "system wave" and so the first sequence in a Fourier Series model of the solar system. This paper now gives a theoretical significance to this "system wave" in the context of the circular restricted Three Body Problem (3BP). The "system wave," being the solution to the Ten Body Problem implies the planets exist in stable positions in the non-rotating problem. In the rotating problem this "system wave" reduces to an arc between Lagrange Points L4 and L5, thereby establishing a Lagrange Continuum with L3 at the center, on the intersection with the x-axis. This continuum of stability points is shown to coincide - and thus be proven by - the asteroid belt.

The Three Body Problem

A proof of the system wave is in the way the Lagrange stability points in the 3BP are found. That is, two circles centered at the primary masses are gradually increased in diameter. The first point where they intersect is L2; the second point L1 (eg the larger mass + the outer zero velocity curve); the third point L3 (eg the smaller mass + the outer zero velocity curve); and L4 and L5 where they finally intersect - both circles + the zero velocity curve. In analytic geometry, intersections of these curves are by definition where the partials are zero; thus they are stability points.

In the following paper it is shown that the significance of "j" is a relative motion between the inner and outer planet regimes. That means the Lagrange Arc from L2 outward to L3/L4 becomes a waveform; similar to the "system wave" derived here. A relative motion between the Lagrange points on the arc, in the non-rotating system, make the arc into a helix. Presumably the arc is twisted into a helix because of a relative motion between the inner and outer planets. The geometry of what is known of the problem can be used to develop an approximate ratio.

The system wave has about 36 wavelengths between the primaries. To make this happen, the relative motion between the inner and outer planet regimes must be 1:36. This is of the same order as the ratio of the average period of the inner planets (in years: 0.24,0.61,1.0,1.88) versus the outer planets (in years: 11,29,84,164).

So, in engineering terms, the inner gear rotates 36 times for every rotation of the outer gear. If L4 and L5 are attached to separate gears, then the Lagrange Arc between them will be twisted into a helical waveform such as the "system wave." A synodic period of three years between the two regimes would cause the Earth-moon cusp to rotate once every twelve months, keeping the same face of the moon toward Earth at all times.

This idea of having a synodic period that is the same as the ratio of rotations is the same as for the Earth-moon system. Consequently, our own solar system is at the cusp of a much larger, integalactic "system wave," perhaps with the outer planetary loops paralleling this huge wave to keep our own solar system synchronous with the whole. Just as you would expect, the rotational period of the sun is the same as the Earth's moon - 27 days - making the whole mechanism operate in sync. Also, the moons' gravitational cycle of 18 years coincides with the Saros; an 18 year cycle in which the exact alignment of the Sun, Earth, and Moon repeats.

It is easy to see that this fits in nicely with the mathematical solution. In the following section the veracity of "j" is established. We have just seen that the "system wave" of our solar system interacts with a much large entity on the galactic scale, whereupon the two wave forms sum to 1 ($\cos^2 + \sin^2$); thus the so-called Szebehely Equation is solved.

The asteroids identify this theory, as they exist in a continuous range of mean motions near the orbit of Jupiter. If you look at the problem in the rotating 3BP based on the invariant plane, then the effect is that L4 and L5 have relative motion as silhouetted by the asteroids.

An interesting aside is that, in the Ten Body sense our solar system is actually Earth-centered, the great majority of the sun's gravity going into establishing the continuity of our solar system in the Milky Way galaxy and leaving the intricate Earth-moon system to determine the order and cohesiveness of the sun's nine satellites.

The Gravity Ellipsoid

Another way to look at the development thus far is possible in the concept of an ellipsoid in 3D space. The 3BP orbital plane is a plane though the "gravity ellipsoid" (similar concepts exist in dynamics for angular momentum and inertia), with the L4 and L5 stability points being on the two principal axes, both of which are stable. One of them is the primary axis (with the largest value) and the other is the secondary axis (with the smallest value) which in this case is the one that moves. The third axis is the "system wave," identified in the ellipsoid as a separatrix. This is not an unstable axis, but the boundary between two stability regions surrounding the primary axes - like the seams on a baseball. This is analagous to, in the 3BP orbital plane, to the colinear axis on which the three unstable Lagrange Points L1, L2 and L3 exist but about which objects can be put in a stable orbit (eg the halo orbits around the L2 and L3 points of the Earth-Moon system).

Thus what began as a line representing a continuity of n stable points; ten of which are already identified by the Sun and Planets, has now been expanded into a three dimensional surface; every line on which is a separate continuum of the "Ten Body Problem." In addition, the two stable axes and the unstable "system wave" axis are integral to a "gravity ellipsoid" which can be thought of as a continuum for the "N Body Problem."